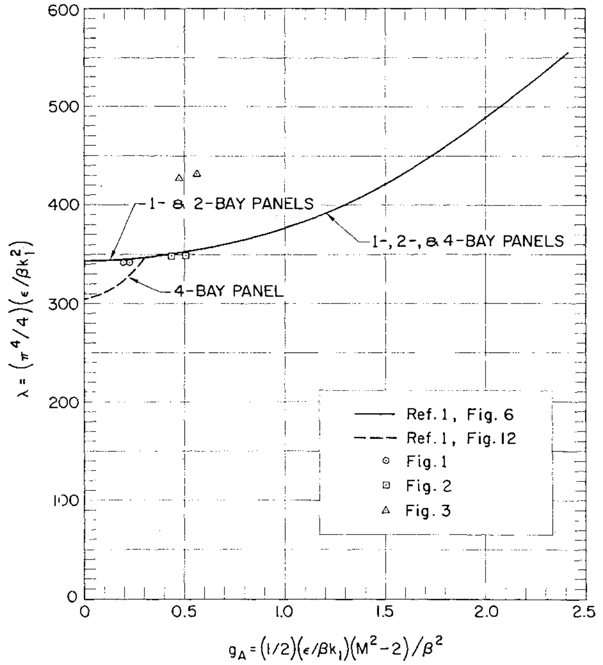

Fig. 4 Comparison of flutter stability boundaries for zero structural damping.

also related to the notation of Ref. 2 through the definitions $\lambda = (\pi^4/4)(\epsilon/\beta k_1^2)$ and $g_A = \frac{1}{2}(\epsilon/\beta k_1)(M^2 - 2)/\beta^2$. The abscissa g_A is shown in Fig. 4 rather than the total damping coefficient g_T of Ref. 1, where $g_T = g_A + g_S$ and g_S is the equivalent viscous structural damping coefficient, because the conventional structural damping can be destabilizing, whereas viscous damping is always stabilizing. The destabilizing effect of structural damping is apparent in Figs. 1–3 from the reversed slope of the critical curves for moderate values of g. This destabilizing effect of structural damping on panel flutter has been analyzed by Johns[10]; certain essentially different properties of viscous and structural dampings were first pointed out by Soroka.[11]

The comparison in Fig. 4 shows excellent agreement between Figs. 1 and 2 and Ref. 1. The 20% discrepancy between Fig. 3 and Ref. 1 measures the convergence error in terms of stiffness requirement of the 16 degree-of-freedom idealization of the four-bay panel. Particularly significant is the agreement between Fig. 1 and Ref. 1, since it indicates that the first-order unsteady aerodynamic theory is valid for panel flutter calculations at supersonic Mach numbers at least as low as $M = 1.56$; the determination of the lower Mach number limit of the first-order unsteady theory is one of the subjects of the current investigation on multibay panels in Ref. 9.

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Shock Interaction in a Hypersonic Flow

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Nomenclature

δ = flow deflection angle
ϵ = angle of the contact surface with respect to the freestream flow direction
P = static pressure
θ = shock wave angle
M = Mach number
δ_b = wedge body angle
α = angle of attack of wedge body
$d\delta^*/dx$ = slope of mass boundary layer
T = static temperature
T_b = wedge body temperature
μ = viscosity
μ_b = viscosity at surface of wedge
Re_x = Reynolds number at point of interaction

Subscripts

0 = freestream conditions
1 = conditions in region 1 as specified by Fig. 1
2 = conditions in region 2 as specified by Fig. 1
3 = conditions in region 3 as specified by Fig. 1
4 = conditions in region 4 as specified by Fig. 1

Introduction

The interaction of oblique shock waves of different intensities in a hypersonic flow was investigated from both the theoretical and experimental viewpoints. The angle, with respect to the freestream flow direction, of the vortex layer or contact surface generated by this interaction phenomena is predicted using first an assumption of inviscid flow and then an assumption of viscous flow. Empirical solutions are also developed to predict the angle of the contact surface with a minimum of computation if a high degree of accuracy is not required. The effect of varying the flow and geometric parameters on the viscous solution was also considered.

Although innumerable reports have been published on the theory of shock waves in a supersonic flow, there appears to be very little work accomplished in the field of interaction of oblique shock waves of different families. A study was con-

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ducted by Molder[1] for a range of steady flow Mach numbers from 2.0 to 5.0. This note, although using a similar approach to that of Molder's, includes the effect of viscosity on the interaction phenomena. The viscous and inviscid solutions are compared and recommendations made as to under what circumstances the inviscid assumption is valid.

Theoretical Analysis

The interaction of oblique shock waves of unequal strength in a steady supersonic flow is characterized by a zone of constant pressure and flow velocity that exist behind the point of interaction. This zone is commonly referred to as a vortex layer or a contact surface.[2] A contact surface has zero flux through it and separates two zones of different Mach number, density, and temperature as it moves with the gas. The static pressure and flow direction on either side of the contact surface are, however, the same, facts that permit the angle of the contact surface with respect to the freestream direction to be computed.

Inviscid solution

Figure 1 illustrates schematically the configuration used to generate the shocks and the resulting shock wave pattern. Noting now the specified conditions of parallel flow and equal static pressure on either side of the contact surface, and analyzing the geometrical considerations of Fig. 1, the following equations may be written:

$$\delta_3 = \delta_1 + \epsilon \tag{1}$$

$$\delta_4 = \delta_2 - \epsilon \tag{2}$$

$$P_3 = P_4 \tag{3}$$

Using a specific heat ratio of 1.40 and the oblique shock relations as expressed in Ref. 3, the following relation may be derived from Eq. (3):

$$\frac{7M_0^2 \sin^2\theta_1 - 1}{7M_0^2 \sin^2\theta_2 - 1} = \frac{7M_2^2 \sin^2\theta_4 - 1}{7M_1^2 \sin^2\theta_3 - 1} \tag{4}$$

The left-hand side of Eq. (4) is readily calculated, as are M_1 and M_2. However, θ_3 and θ_4 are functions of δ_3 and δ_4, which are functions of ϵ. Thus, an iterative solution is required to solve Eq. (4).

A solution was programed on the IBM 7090 to iterate on ϵ until a value was arrived at that was accurate to 0.0001°. In all cases considered, it was found that this accuracy of ϵ yielded a difference between the right- and left-hand sides of Eq. (4) of less than 10^{-5}.

Viscous solution

Since the viscous solution is, in general, very complex, certain simplifying assumptions were made to reduce the necessary computation. The effects of these assumptions on the parameters under consideration are negligible for this analysis. As shown by Lees and Probstein, dissociation and other nonequilibrium effects have little or no significance at the temperatures under consideration.[4] Ivey and Cline investigated the effect of variable heat capacity on the flow through oblique shock waves and found it negligible.[5] In addition, a uniform flow field will be assumed which permits use of Eq. (4).

Using the assumption that the boundary-layer thickness is the same as the displacement thickness, the new effective flow deflection angle may be expressed as

$$\delta = \delta_b + \alpha + (d\delta^*/dx) \tag{5}$$

Thus, the viscous problem is reduced to a solution similar to the inviscid case once the value of $d\delta^*/dx$ at any particular location on the body is determined.

The method of Lees and Probstein was used to determine this boundary-layer shock wave interaction.[6] Using the zero pressure gradient solution for a boundary layer on an inclined wedge derived by Crocco, with a linear viscosity temperature law, and assuming a constant Prandtl number of 0.725, the following relation may be derived:

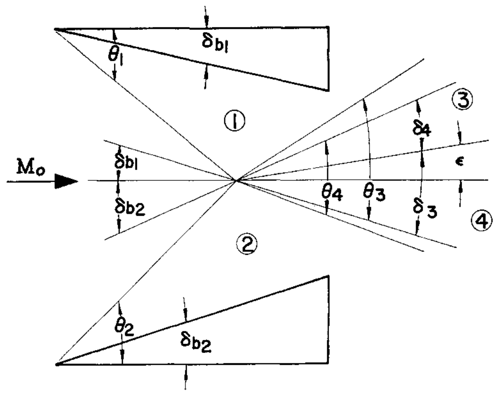

Fig. 1 Schematic diagram of the geometrical relations between the flow deflection angles, shock angles, and the contact surface angle.

$$\frac{d\delta^*}{dx} = \left[\frac{0.968\, T_b}{M^2 T} + 0.058\right]\left[\frac{\mu_b}{\mu}\frac{T}{T_b}\right]^{1/2}\frac{M^2}{Re_x^{1/2}} \tag{6}$$

where the Reynolds number is based on the inviscid conditions behind the shock at the point of interaction.

Initially a value of zero was assumed for $d\delta^*/dx$ in Eq. (5) and the freestream conditions behind the shock calculated per Ref. 3. Knowing these conditions, Eq. (6) may now be solved and a new value of $d\delta^*/dx$ found. This procedure was repeated until two successive values of $d\delta^*/dx$ differed by less than 10^{-4}. Having now a value of $d\delta^*/dx$, the solution is identical to the inviscid one.

Empirical analysis

Since the time to solve this problem on even the highest speed digital computers is relatively great, an empirical method was developed to predict ϵ with a fair degree of accuracy and a minimum of computation. The relations derived for the stated Mach number ranges are shown below:

$$6 \leqslant M_0 \leqslant 11:\ \epsilon = \frac{29.2\delta_2 + 914.75}{\delta_1 + 31} + \frac{3\delta_2}{100M_0}(\delta_2 - \delta_1) - 29.14 \tag{7}$$

$$11 \leqslant M_0 \leqslant 17:\ \epsilon = \frac{29.2\delta_2 + 914.75}{\delta_1 + 31} + (16 - M_0)(\delta_2 - \delta_1 - 2)10^{-2} - 29.14 \tag{8}$$

$$17 \leqslant M_0 \leqslant 20:\ \epsilon = \frac{29.2\delta_2 + 914.75}{\delta_1 + 31} - \delta_2(\delta_2 - \delta_1 + 2)(0.106M_0 - 0.63235)10^{-3} - \frac{19 - \delta_2}{(22 - M_0)25} - 29.14 \tag{9}$$

Theoretical Results

The three methods described previously for finding ϵ were compared for a range of Mach numbers from 6 to 20 and for a range of wedge body angles between 2° and 30°.

For Mach numbers between 6 and 12, the variation between the inviscid and viscous solutions for ϵ is relatively slight, with the greatest differences appearing when the δ_b's are small. The empirical solution on the other hand tends to be in error by the greatest amount when the ratio of δ_{b_2} to δ_{b_1} is on the order of 0.5 or for the large values of δ_{b_2} when δ_{b_1} is small. At Mach numbers above 12, the viscous and inviscid solutions for ϵ begin the differ by appreciable amounts for small δ_b's.

However, the inviscid method is still accurate to within 10% if the wedge angles are greater than 8° and $M_0 < 16$. The trend of the empirical solution for $M_0 > 12$ shows that its maximum deviation from the viscous solution for ϵ occurs at the minimum values of δ_{b_1}, with the intermediate values of δ_{b_1} yielding the minimum differences between the two solutions.

It was also found that the variation of ϵ between the inviscid and viscous solution was a maximum for the combination of high wedge body temperature, low total pressure, high Mach number, and shock interaction location near the leading edge. The effect of total temperature was found to be negligible.

A more complete analysis of the data obtained in the comparison of the three methods of solution may be found in Ref. 7.

Experimental Results

The experimental portion of this investigation was conducted in the 20-in. hypersonic wind tunnel located in the Aerospace Research Laboratories at Wright-Patterson Air Force Base, Ohio. A complete description of the facility and its calibration data may be found in Ref. 8.

The tests to obtain this shock interaction data were conducted at a nominal freestream Mach number of 14.36 and total pressure and total temperature conditions of 1600 psia and 1850°R, respectively. The freestream Reynolds number per foot was 0.729×10^6. The range of wedge body angles used in the experimental program included angles from 11.95° to 27.33°.

Since there is a density gradient across the contact surface, it should be visible to schlieren observation. However, because of the heat conduction between the permanently adjacent particles on either side of the discontinuity, the contact surface layer will tend to dissipate rather rapidly. Thus, it was considered improbable at the outset of the tests that this region of discontinuity would be visible. Subsequent testing verified this assumption, although in a few cases this surface was visible and measurable. However, since the shock angles were visible, this information could be used to find δ_3 and δ_4 from Eq. (1) or (2). However, because of possible errors in measuring the shock wave angles, it is evident that the values of ϵ computed may not exactly agree. In general, however, it was found that the values of ϵ agreed to measurable accuracy.

Using the measured values of θ_3 and θ_4 to compute ϵ, it was found that the average error between this value and the value of ϵ from the viscous solution was 7.1%, with the maximum error encountered being 34.3%. In the cases where the contact surface was visible to schlieren observation, the average error between the viscous solution for ϵ and the measured value was 7.5%, with the maximum error being 19.4%.

Conclusions

In general, it was found that the inviscid solution is an accurate method for predicting the contact surface angle for all wedge angles at hypersonic Mach numbers below 10. Above Mach 10, however, some care must be exercised in using the inviscid solution, especially for small wedge angles. For values of wedge body angles above 8°, however, the inviscid solution is fairly accurate up to a Mach number of 16. Beyond Mach 16, it is not recommended to use the inviscid solution for any values of wedge angles.

The empirical equations developed to predict ϵ are highly accurate for Mach numbers below 10, considering the vast amount of calculations that they save. At high Mach numbers, however, care should be exercised in their use, just as for the inviscid solution. The empirical equations are, however, slightly more accurate for the higher Mach numbers than the inviscid solution.

Analyzation of the experimental data shows that the viscous solution for predicting the angle of the contact surface with respect to the freestream flow agrees quite well with the experimental data for a Mach number of 14.36. The errors that are present may well be attributed to inaccuracies in measuring the shock wave angles. Hence, it would appear from these results that the viscous method as developed in this note for predicting ϵ is valid for any hypersonic Mach number, at least up to the value used in the experimental portion of these tests.

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Addendum: "The Vertical Water-Exit and -Entry of Slender Symmetric Bodies"

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Nomenclature

C_F = upward thrust coefficient (net upward force/maximum frontal area/freestream dynamic pressure)
Fr = square of Froude number (U^2/gl)
g = acceleration due to gravity
l = body length
$S(y)$ = body cross-sectional area, $\pi X^2(y)$†
U = body speed
$X(y)$ = body radius†
x = radial coordinate†
y = axial coordinate,† measured downward from upper end of body
Z = displacement of upper end of body above undisturbed position of free surface†
ϕ = velocity potential, made dimensionless with Ul
τ = body thickness ratio, maximum diameter/length

Introduction

THE purpose of this note is to remove certain limitations of the solution derived in Ref. 1, viz., its inapplicability to blunt-ended bodies of revolution and its restriction to large but finite Froude number situations.

Treatment of Blunt-Ended Bodies of Revolution

When applied to round-ended bodies, such as the ellipsoid of revolution, the formal slender-body theory of Ref. 1 pre-

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† X, x, y, and Z are dimensionless, normalized by the body length l.